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A FAST ALGORITHM FOR THE CALCULATION OF NEUTRON SELF-SHIELDING FACTORS

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Abstract: Thermal neutron self-shielding factors are calculated for different materials used in reactor dosimetry. The effect of multiple scattering inside the probe is taken into account within a new theoretical formalism recently proposed |1| with application to foils. Here, the method is extended to probes of cylindrical geometry. A fast algorithm is developed for the calculation of the self-shielding based on an approximation equivalent to nearly six collisions inside the probe, which gives quite satisfactory results in the thermal region.

(self-shielding, scattering, thermal neutrons, wires, foils)

Introduction

The effect of multiple scattering on neutron self-shielding factors was considered in |1| with in a new theoretical model which present some advantages if compared with the formal treatment based on the transport theory |2,3|. The model was applied to the calculation of thermal self-shielding factors in foils, for which the code FOILTH was constructed. In this paper, the formalism is applied to wires for which a fast algorithm is developed (code CYLKCT). The base of the algorithm is an approximation equivalent to consider nearly six sucessive collisions of neutrons inside the probe. However, the information required by this method is just the same as that of the second-collision approximation.

Tables are presented with thermal self-shielding factors for different materials with different dimensions.

Application of the model to wires

According to the model published in |1| the absorption probability in a probe of arbitrary symmetry can be written as the integral over its volume

$$\varphi = \int \Psi_0 (r)\, K(r)\, dV \qquad (1)$$

where $\Psi_0 (r)$ is the first-collision probability density (which depends on the external field) and K(r) is the number of collisions which a neutron suffers inside the probe after having been scattered at the point r of its first collision. Equation (1) expresses an important feature of the present formalism. It is based on the new collision function K which does not depend on the characteristics of the external field but is exclusively determined by the geometry and the collision properties of the probe. Once the integral equation for K is calculated, it can be applied to any external field by changing Ψ_0.

For isotropic neutron fields the integral (1) can be simplified making use of the symmetry of the probe. For wires the problem is reduced to one dimension using the variable ρ which characterizes the depth inside the cylindrical probe. As the collision probability is constant over the cylindrical surface of radius ρ, the functions Ψ_0 and K depend only on ρ, and the self-shielding factor G can be written as:

$$G = \frac{1}{2\Sigma R} \int_0^R \Psi_0(\rho)\, K(\rho)\, d\rho \qquad (2)$$

where R is the radius of the wire and Σ the macroscopic cross-section. It should be noticed that even when the external field is not isotropic the collision function K remains depending only on ρ. The case is not the same with Ψ_0.

The function $\Psi_0 (\rho)$ for wires was calculated in |4| where the second-collision approximation was used to consider the effect of scattering on thermal neutron self-shielding factors.

The integral equation for K assumes the form

$$K(\rho) = 1 + \frac{\Sigma s}{\Sigma} \int Q(\rho,\rho')\, K(\rho')\, d\rho' \qquad (3)$$

where Σs is the macroscopic scattering cross-section and $Q(\rho,\rho')$ is the probability density for a neutron being scattered at the depth ρ to interact again at ρ'.

The solution of this equation by sucessive iteractions starting with K=1, leads to the sucessive multiple collision approximations. The convergence of this method may be very slow depending on the cross-section of the material. Besides, this method is troubled by the kern $Q(\rho,\rho')$ which has a very complicated structure. A fast solution however may be obtained in a good approach following the ideas expressed in |1|. Considering that $Q(\rho,\rho')$ has a maximum at point ρ'=ρ we can expect that the function K(ρ) does not vary too much from the center till the surface of the probe. This allows us to take K(ρ') off the integral (3) at the point ρ'=ρ obtaining, in first approximation:

$$K_c(\rho) = \frac{1}{1 - U(\rho)} \qquad (4)$$

where

$$U(\rho) = \frac{\Sigma s}{\Sigma} \int Q(\rho,\rho')\, d\rho'$$

The integral of $Q(\rho,\rho')$ represents the probability of a second interaction inside the probe and can be expressed through the escape probability W(ρ) from the depth ρ. The escape probability W(ρ) was studied in |4| and its calculation is

much easier than that of $Q(\rho,\rho')$.

Thus, eq.(4) takes the form:

$$K_c(\rho) = \frac{1}{1 - \Sigma s/\Sigma(1-W(\rho))} \qquad (5)$$

From the expression of K corresponding to the second collision approximation,

$$K_2 = 1 + \frac{\Sigma s}{\Sigma}(1 - W(\rho))$$

it can be seen that the calculation of G within the approximation $K=K_c$, requires the same functions Ψ_o and W as the second collision treatment does. However, the approximation $K=K_c$ is much better than that of $K=K_2$. It was studied in foils [1] and the conclusion was that the approximation $K=K_c$ is equivalent to consider at least five or six collisions inside the foils, which seems to be quite reasonable in most of the cases.

The algorithm

The K_c approximation, which proved to be very accurate in foils, is the base of our fast algorithm for the thermal neutron self-shielding calculation in wires. An illustration of this function is shown in Fig. 1, where K_c is plotted as a function of the parameter ρ/R for several values of ΣR. For each value of ΣR there are two curves with different ratios $\Sigma s/\Sigma$, showing the effect of scattering.

When ΣR is large, the function is almost constant except close to the surface where K_c decreases. This effect is enhanced when scattering is important. When ΣR is small, K_c is close to unity at all depths and the influence of the ratio $\Sigma s/\Sigma$ decreases.

A further step in constructing a fast algorithm to calculate G, using the eqs. (2) and (5), was a special integration subroutine AINT for monotonously varying functions as it is the case of Ψ_o.

In Fig.2 the function $R\,\Psi_o$ is plotted versus ρ/R for different values of the parameter ΣR.

All the curves go to zero at the center of the cylinder i.e. $\rho/R=0$. This fact is related with the linear decrease of the area of the cylindrical surfaces with the radius ρ.

When ΣR is small, the probe material is in some degree transparent to neutrons and, as one could expect, the collision probability density is very low and linear.

For large values of ΣR, neutrons only penetrate into a small region close to the surface where a strong interaction takes place.

Because of the behaviour of $K_c(\rho)$ the function $\Psi_o K_c$, which must be integrated over the parameter ρ, has almost the same curvature as Ψ_o.

The integration subroutine AINT uses a variable number of pivotal points, depending on the curvature of the functions, distributing them according to the required precision. The number of points is the minimum, within the method, which is very important as both functions Ψ_o and W involve the calculation of a long time consuming double integral over the solid angles (see [4]).

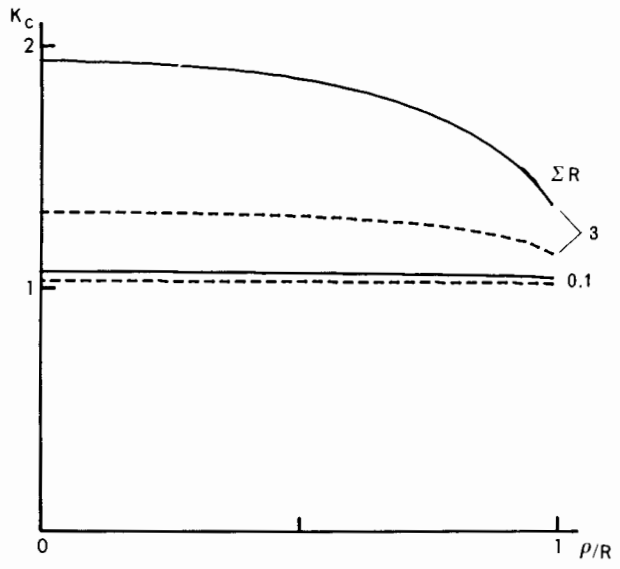

Fig. 1 K_c versus ρ/R for two values of ΣR. The continuous lines correspond to a ratio $\Sigma s/\Sigma$ of 0.5 while for the dashed ones this ratio is 0.25.

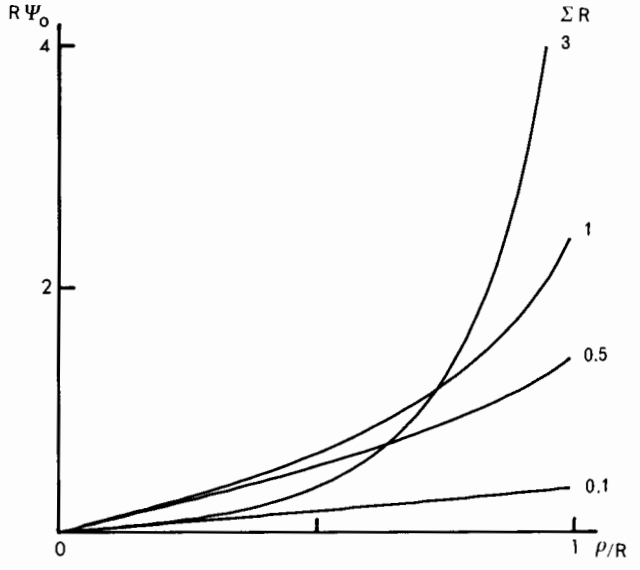

Fig. 2 The function $R\Psi_o$ versus ρ/R for different values of ΣR.

Results

In reactor dosimetry, different kinds of detectors are used as neutron flux monitors, namely thin foils and self-powered neutron detectors (SPND) with emitters commonly of cylindrical geometry. Thus, here, we present self-shielding factors calculated both for thin foils and for wires.

The used macroscopic cross-section are presented in Table 1. |5,6|.

In Table 2., values of G are shown for wires of materials commonly used as emitters in SPNDs.

The effect of multiple scattering for the presented radia is reduced because of the small thermal cross-sections of the materials and the second-collision approximation works quite well (namely for R<0.1 cm). Even in the case of vanadium, where the ratio $\Sigma s/\Sigma$ is high, the mean free path is so large that the self-shielding factor itself represents a small correction and again two or three collisions are enough.

For materials commonly used as activation detectors we present Table 3., with self-shielding factors both for thin foils and for cylindrical probes with radius equal to the foil thickness. These results where respectively obtained with code FOILTH and code CYLKCT. The first one calculates G with the exact collision function K, in foils.

One can observe that the self-shielding factors of dysprosium assume rather small values for thicknesses greater than 0.01 cm which is determined by its absorption cross-section. Again as $\Sigma s/\Sigma$ is small, the second collision approximation is quite accurate for smaller thicknesses but for higher ones it is necessary the K_c approximation.

The case of copper is similar to that of vanadium (Table 2.).

Table 1. Macroscopic-cross sections used in this paper |5,6|

MATERIAL	MACROSCOPIC CROSS SECTIONS $\|cm^{-1}\|$		$1/\Sigma$
	Σs	Σa	$\|cm\|$
Co	0.546	3.381	0.255
Ho	0.277	2.074	0.425
Er	0.293	5.184	0.183
Hf	0.463	4.675	0.195
V	0.347	0.367	1.401
Ag	0.298	3.711	0.250
Cu	0.661	0.321	1.019
Dy	3.352	29.75	0.030
Au	0.463	5.827	0.159
In	0.094	7.430	0.133
Mn	0.177	1.069	0.803
Sc	0.897	1.089	0.504
Fe	0.964	0.217	0.847

Table 2. Self-shielding factors for wires of materials commonly used as emitters in SPNDs, in the Kc approximation.

R $\|cm\|$	Co	Ho	Er	Hf	V	Ag
0.005	0.978	0.987	0.967	0.970	0.998	0.976
0.01	0.958	0.974	0.937	0.943	0.996	0.954
0.02	0.920	0.949	0.882	0.892	0.991	0.912
0.03	0.885	0.926	0.833	0.847	0.987	0.874
0.04	0.852	0.903	0.788	0.805	0.982	0.839
0.05	0.821	0.882	0.747	0.766	0.978	0.806
0.06	0.791	0.862	0.709	0.731	0.973	0.775
0.07	0.764	0.842	0.675	0.698	0.969	0.746
0.08	0.738	0.823	0.643	0.667	0.965	0.719
0.09	0.713	0.805	0.614	0.639	0.961	0.694
0.1	0.690	0.788	0.586	0.612	0.956	0.669
0.11	0.668	0.771	0.561	0.588	0.952	0.647
0.12	0.647	0.755	0.537	0.564	0.948	0.625
0.13	0.627	0.739	0.515	0.543	0.944	0.605
0.14	0.608	0.724	0.495	0.522	0.940	0.585
0.15	0.590	0.709	0.476	0.503	0.936	0.567
0.16	0.573	0.695	0.458	0.485	0.932	0.550
0.17	0.557	0.681	0.441	0.468	0.928	0.533
0.18	0.541	0.667	0.425	0.452	0.924	0.517
0.19	0.526	0.654	0.410	0.437	0.920	0.502
0.20	0.512	0.642	0.396	0.423	0.916	0.488
0.21	0.498	0.630	0.383	0.409	0.912	0.474
0.22	0.485	0.618	0.370	0.396	0.908	0.461
0.23	0.472	0.607	0.358	0.384	0.904	0.449
0.24	0.460	0.595	0.347	0.373	0.900	0.437
0.25	0.449	0.585	0.336	0.362	0.897	0.426

Table 3. Self-shielding factors for foils and wires with radius equal to the corresponding foil thickness. The first line corresponds to foils (code FOILTH) and the second one to wires (code CYLKCT).

δ=R $\|cm\|$	Co	Cu	Dy	Au	In	Mn	Sc
0.0005	0.994	0.999	0.963	0.991	0.988	0.998	0.998
	0.998	1.000	0.981	0.996	0.995	0.999	1.000
0.001	0.989	0.999	0.935	0.983	0.979	0.996	0.996
	0.996	1.000	0.963	0.992	0.990	0.999	0.999
0.002	0.981	0.998	0.890	0.970	0.962	0.993	0.993
	0.991	1.000	0.929	0.985	0.981	0.997	0.998
0.003	0.973	0.997	0.852	0.958	0.948	0.990	0.991
	0.987	0.999	0.897	0.978	0.971	0.996	0.996
0.004	0.966	0.996	0.819	0.947	0.934	0.987	0.988
	0.983	0.999	0.867	0.970	0.962	0.995	0.995
0.005	0.959	0.995	0.789	0.936	0.922	0.984	0.985
	0.978	0.999	0.839	0.963	0.954	0.993	0.993
0.006	0.953	0.995	0.762	0.926	0.910	0.982	0.983
	0.974	0.998	0.812	0.956	0.945	0.992	0.992
0.007	0.946	0.994	0.737	0.917	0.899	0.979	0.981
	0.970	0.998	0.787	0.950	0.937	0.990	0.991
0.008	0.940	0.993	0.713	0.908	0.889	0.977	0.978
	0.966	0.997	0.763	0.943	0.928	0.989	0.989
0.009	0.935	0.992	0.691	0.900	0.879	0.975	0.976
	0.962	0.997	0.740	0.936	0.920	0.988	0.988
0.01	0.929	0.992	0.671	0.891	0.869	0.972	0.974
	0.958	0.997	0.719	0.930	0.912	0.986	0.987
0.02	0.880	0.985	0.517	0.821	0.787	0.951	0.953
	0.920	0.993	0.548	0.869	0.839	0.973	0.974
0.03	0.839	0.978	0.418	0.764	0.722	0.932	0.936
	0.885	0.989	0.435	0.815	0.775	0.960	0.961
0.04	0.803	0.972	0.347	0.716	0.669	0.915	0.920
	0.851	0.985	0.356	0.767	0.719	0.948	0.948
0.05	0.771	0.967	0.295	0.674	0.623	0.899	0.905
	0.820	0.981	0.299	0.723	0.670	0.935	0.936
0.1	0.646	0.942	0.150	0.522	0.463	0.833	0.840
	0.690	0.963	0.161	0.555	0.490	0.879	0.879

Table 4. Comparison between the results of codes FOILTH and CYLKCT with experimental and other theoretical values.

	FOILS				WIRES			
MATERIAL	δ [cm]	$G_{calc.}$ [7]	$G_{exp.}$ [8]	G (FOILTH)	R [cm]	G_{calc} [9]	$G_{exp.}$ ([10]), [8]	G (CYLKCT)
Co	0.0052	0.959	0.960±0.01	0.958	0.0127	0.956	(0.940±0.01)	0.948
	0.0102	0.931	0.930±0.01	0.928	0.01905	0.936	(0.920±0.02)	0.923
	0.0152	0.907	0.910±0.008	0.902	0.0254	0.917	(0.900±0.02)	0.900
	0.0204	0.884	0.880±0.01	0.878	0.03175	0.899	(0.880±0.03)	0.879
Cu	0.02	0.983	0.980±0.005	0.984	0.1975		0.915±0.013	0.928
	0.05	0.966	0.965±0.005	0.967	0.2425		0.905±0.008	0.912
	0.1	0.937	0.940±0.006	0.942	0.285		0.895±0.01	0.898
	0.161	0.901	0.918±0.005	0.916	0.355		0.870±0.01	0.874
	0.2	0.881		0.900	0.400		0.855±0.01	0.859
Fe	0.05	0.973	0.978±0.005	0.978	0.245		0.935±0.01	0.912
	0.1	0.943	0.956±0.006	0.962	0.2875		0.923±0.008	0.897
	0.15	0.913	0.941±0.006	0.947	0.3638		0.910±0.01	0.871
	0.2015	0.884	0.938±0.006	0.933	0.4175		0.888±0.005	0.853
					0.4975		0.883±0.005	0.826

In Table 4. the results of codes FOILTH and CYLKCT are compared with experimental data and theoretical values from codes PERTURB-D [7] and PERTURB-C [9].

The referred experimental values for cobalt foils and also copper and iron foils and wires, were obtained not by the usual activation technique, used in [10], but by phase oscilation [8].

The agreement between experimental foil data and the results of code FOILTH is remarkable.

In reference [7] the self-shielding factors are calculated in second-collision approximation with averaged functions over the Maxwellian flux distribution. They are quite close to the experimental data when the scattering cross-section is small compared with the absorption one, as it is the case of Co. However, in copper and iron they underestimate the true value of G.

For wires the values of G obtained with our code CYLKCT agree quite well with the experimental results for Co and Cu. However, for iron, where the effect of scattering is very important, the approximation K_c used in the present work is not enough to obtain the experimental value for such high thick wires. Here, we have to mention that CYLKCT is a code for infinite long wires and discrepancies are expected for finite thick cylinders.

Conclusions

The thermal self-shielding factors of wires and thin foils submitted to an isotropic neutron field were calculated considering the effect of multiple scattering.

The results are presented within a wide range of thicknesses and radia for materials used in reactor dosimetry.

From the comparison between our results and both experimental and other theoretical values, we conclude that for $\Sigma s > \Sigma a$ and large dimension probes, a multiple scattering treatment is required.

For small ΣR or $\Sigma\delta$, however, the second-collision approximation is quite correct.

An extension of the model to the resonance energy region, where scattering can play an important role is now under development.

The numerical results presented here, were obtained with two Fortran codes, CYLKCT for wires and FOILTH for foil, using very fast algorithms. These codes are available on request.

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